Girraween High School

2019 Year 12 Mathematics
HSC Trial Examination

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Calculators and ruler may be used
- All necessary working out must be shown

Total Marks - 100

- Attempt all questions
- Marks may be deducted for careless or badly arranged work

Question 1 (1 mark)

What is the correct value of $|-1|^3 - |-1|$?

A. -2 B. -1 C. 0 D. 1

Question 2 (1 mark)

Which of the following is 0.02534 corrected to 3 significant figures?

A. 0.03 B. 0.0252 C. 0.0250 D. 0.0253

Question 3 (1 mark)

What are all the solutions to the equation $x(x-1) = 6$?

A. $x = 2$ & $x = 3$ B. $x = -2$ & $x = 3$ C. $x = 2$ & $x = -3$ D. $x = -2$ & $x = -3$

Question 4 (1 mark)

Which is the correct factorisation of $27x^3 + 1$?

A. $(3x-1)(9x^2+6x+1)$ B. $(3x+1)(9x^2-6x+1)$ C. $(3x-1)(9x^2+3x+1)$
D. $(3x+1)(9x^2-3x+1)$

Question 5 (1 mark)

Which of the following is the domain of the function $f(x) = \dfrac{2}{\sqrt{1-x}}$?

A. $x > 2$ B. $x < 2$ C. $x > 1$ D. $x < 1$

Question 6 (1 mark)

Given that $\sin\theta = \dfrac{20}{29}$ and $\dfrac{\pi}{2} \le \theta \le \pi$, what is the value of $\cos\theta$?

A. $\dfrac{21}{29}$ B. $-\dfrac{21}{29}$ C. $\dfrac{21}{20}$ D. $-\dfrac{21}{20}$

Question 7 (1 mark)

What is the angle of inclination of the line $x + \sqrt{3}y - 2\sqrt{3} = 0$ with respect to the positive x axis?

A. $30°$ B. $60°$ C. $120°$ D. $150°$

Question 8 (1 mark)

If $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, what is the value of $\log_a 18$?

A. 1.8 B. 1.6 C. 3.0 D. 0.74

Question 9 (1 mark)

The value of $\lim\limits_{t\to\infty} \dfrac{t}{2t+3}$ is

A. 0 B. $\dfrac{1}{5}$ C. $\dfrac{1}{2}$ D. 1

Question 10 (1 mark)

A fair coin is tossed three times. What is the probability of obtaining exactly one head?

A. $\dfrac{1}{8}$ B. $\dfrac{1}{3}$ C. $\dfrac{3}{8}$ D. $\dfrac{1}{2}$

Question 11 (15 marks)

(a) Evaluate $\dfrac{e^2+1}{\pi}$ to two decimal places. [1]

(b) Convert $\dfrac{8\pi}{3}$ radians to degrees. [1]

(c) Solve $2\sin\theta - 1 = 0$ for $0 \le \theta \le 2\pi$. [1]

(d) Express $\dfrac{3+\sqrt{2}}{6+\sqrt{2}}$ with a rational denominator. [2]

(e) Find the values of x for which $|2x-1| < 3$ [2]

(f) Differentiate the following with respect to x

i. $y = x^2 e^x$ [2]

ii. $y = \dfrac{\ln x}{x}$ [2]

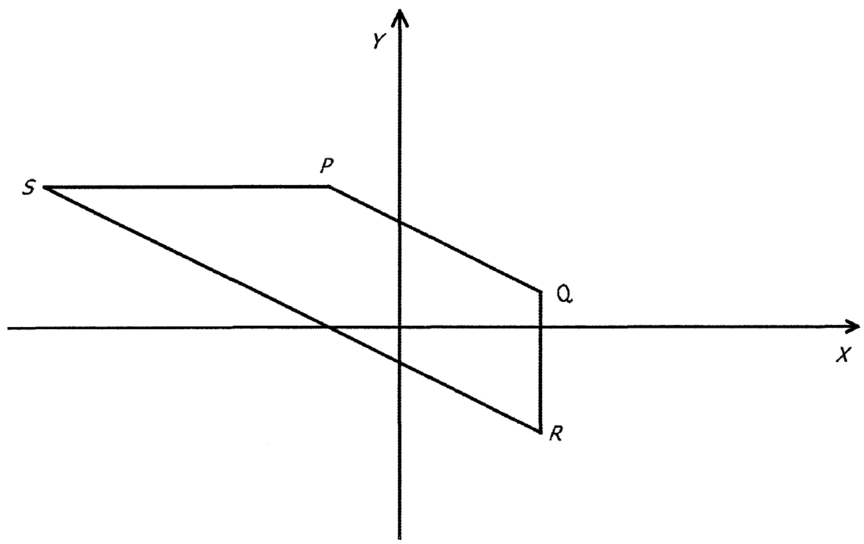
(g) Find $\displaystyle\int \frac{x}{x^2-1}\,dx$ [2]

(h) Find the value of $\displaystyle\sum_{k=0}^{10} 3k+1$ [2]

The exam continues on the next page

Question 12 (15 marks)

(a) In the quadrilateral $PQRS$ the coordinates of the points P and Q are $(-2, 4)$ and $(4, 1)$ respectively. The equation of the line SR is $x + 2y + 2 = 0$.



i. Prove that $PQ || RS$. [2]

ii. Find the length of PQ in exact form. [2]

iii. The line QR is parallel to the y axis, find the coordinates of point R. [2]

iv. Find the perpendicular distance from P to the line RS. [2]

v. If the length of RS is $\sqrt{85}$ units find the area of the quadrilateral $PQRS$. [2]

(b) Consider the parabola given by the equation $y = 8x^2 + 32x + 36$.

i. Show that the equation of the parabola can also be written as $y - 4 = 8(x+2)^2$. [2]

ii. State the coordinate of the vertex of the parabola. [1]

iii. State the coordinate of the focus of the parabola. [1]

iv. State the equation of the directrix of the parabola. [1]

The exam continues on the next page

Question 13 (15 marks)

(a) Consider the curve given by the equation $y = -x^3 + 12x + 1$.

i. Find the coordinates of the stationary points and determine their nature. [3]

ii. Find any points of inflexion. [2]

iii. Sketch the curve, showing the stationary points and any points of inflexion. Do not find the x intercepts. [2]

iv. For what values of x is the curve increasing and concave down? [1]

(b) Use Simpson's rule with 5 function values to find an approximation to the value of [4]

$$\int_0^8 xe^x \, dx.$$

Give your answer correct to one decimal place.

(c) A set of timber logs are stacked in layers. Each layer contains two log less than the layer below. There are five logs in the top layer, seven logs in the next layer, and so on. There are n layers altogether.

i. Find the number of logs in the bottom layer in terms of n. [1]

ii. If there is a total of 140 logs in the whole stack, find the value of n. [2]

The exam continues on the next page

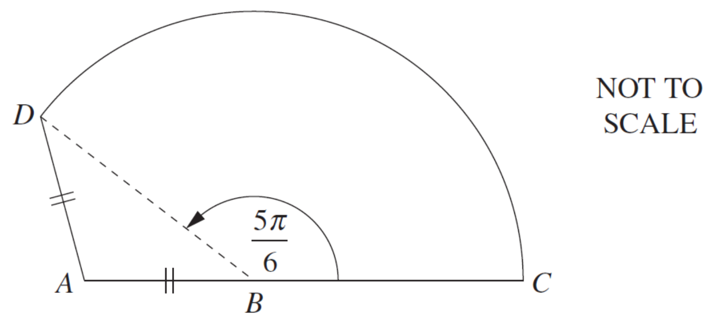
Question 14 (15 marks)

(a) Consider the geometric series $5 + 10x + 20x^2 + 40x^3 + \cdots$

i. For what values of x does the series have a limiting sum? [1]

ii. The limiting sum of the series is 100. Find the value of x. [2]

(b) In the diagram below $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \dfrac{5\pi}{6}$. The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

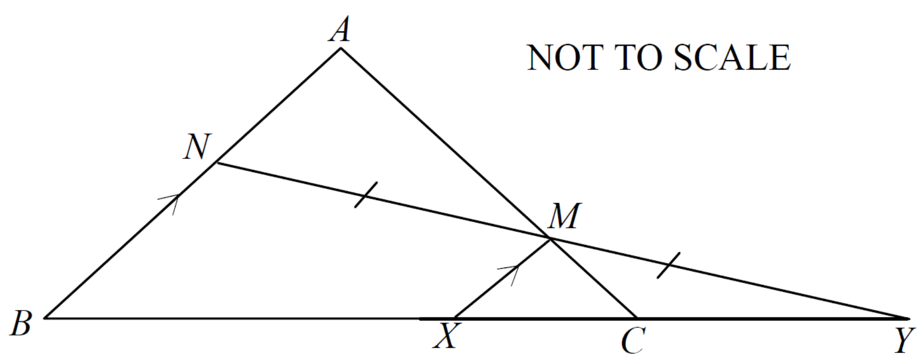


i. Show that $\angle DAB = \dfrac{2\pi}{3}$. [1]

ii. Find the length of BD. [2]

iii. Find the area of the garden $ABCD$ correct to one decimal place. [3]

(c) In the diagram below $\triangle ABC$ is isosceles, with $AB = AC$. The point M is the midpoint of the line NY with $XM || AB$.



i. Prove that $\triangle XMY ||| \triangle BNY$. [3]

ii. Prove that $2MX = NB$. [2]

iii. State the value of $\dfrac{MC}{NB}$. [1]

The exam continues on the next page

Question 15 (15 marks)

(a) Kara borrows \$2500 at an interest rate of 12% p.a. compounded monthly. Kara wishes to pay off this loan with monthly repayment of \$50. let $\$A_n$ be the amount owing at end of the n^{th} month.

i. Show that $A_2 = 2500(1.01)^2 - 50(1 + 1.01)$ [1]

ii. Find how many months it would take for Kara to pay off her loan. [2]

iii. Find the total interest paid by Kara. [1]

iv. Find how many months it would take for Kara to pay off her loan if the interest reduced to 9% p.a. [2]

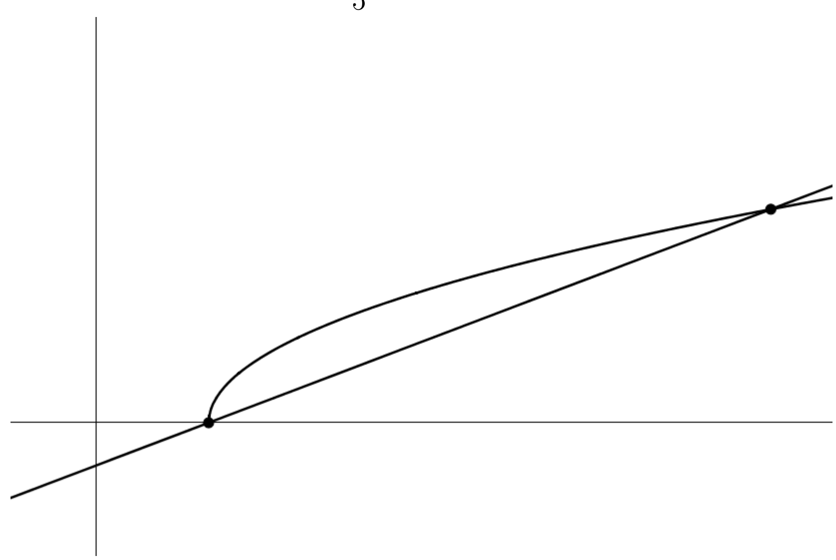
(b) The population of a certain insect is growing exponentially according to the equation $P = 50e^{kt}$, where t is the time in days after the insects are first counted. After four days the population has doubled from the initial amount.

i. Show that $k = \dfrac{1}{4}\ln 2$ [2]

ii. At what rate is the population increasing at day 10? [2]

iii. How long will it take for the number of insects to be 1000? [2]

(c) The graphs of $y = \sqrt{x-5}$ and $y = \dfrac{1}{5}(x-5)$ are given in the diagram below.

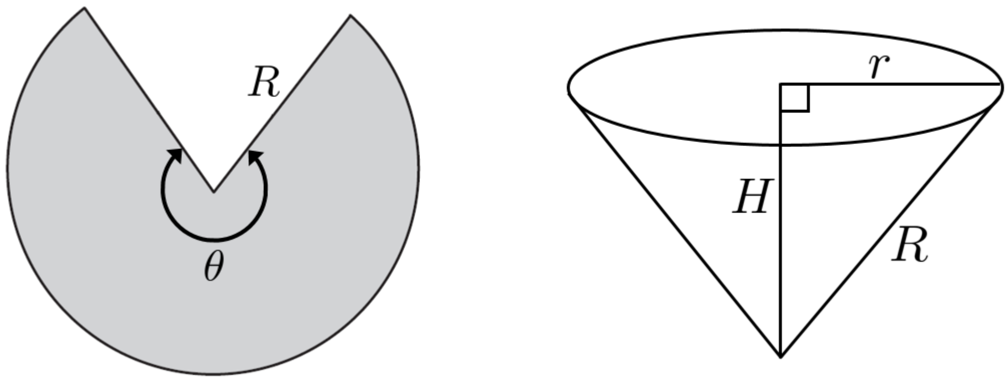


i. The two graphs intersect at two points, find the x coordinates of these two points. [1]

ii. Find the area between the two graphs between the two intersections. [2]

Question 16 (15 marks)

(a) From a circular disc of fixed radius R, a sector subtending an angle of θ at the centre is cut out, where $0 < \theta < 2\pi$. This sector is used as a net to form a cone with radius r and height H as shown below. The volume of this cone V is given by $V = \dfrac{1}{3}\pi r^2 H$.



i. Show that $r = \dfrac{R\theta}{2\pi}$. [1]

ii. Show that $H^2 = \dfrac{R^2}{4\pi^2}(4\pi^2 - \theta^2)$. [1]

iii. Show that $V = \dfrac{R^3\theta^2}{24\pi^2}\sqrt{4\pi^2 - \theta^2}$. [1]

iv. Find the exact value of θ when $\dfrac{dV}{d\theta} = 0$. [3]

(b) The velocity of a particle travelling along the x-axis is given by $v = (t-3)e^{-t}$.

i. State all the possible times for which $v < 0$. [1]

ii. Show that $v = \dfrac{d}{dt}(2-t)e^{-t}$. [1]

iii. Let T be a time such that $T > 3$. Show that the distance travelled by the particle when $t = T$ is given by [2]

$$2 + \frac{2}{e^3} + \frac{2-T}{e^T}.$$

iv. State the eventual distance travelled by the particle. [1]

(c) The coefficients a, b and c of the quadratic $y = ax^2 + bx + c$ are randomly chosen from the set of ten integers $\{-5, -4, -3, -2, -1, 1, 2, 3, 4, 5\}$, with repetition allowed. For example if $a = 1$, $b = 1$ and $c = 2$ then the quadratic becomes $y = x^2 + x + 2$.

i. How many different quadratics can be formed? [1]

ii. Find the probability that the quadratic has two roots α and β such that [1]

$$\frac{\alpha + \beta}{\alpha\beta} = 1$$

iii. Find the probability that the quadratic has two roots α and β such that $\alpha = \beta$. [2]

2019 2U HSC TRIAL.

CDBDD BDBCC

Q1

$|-1|^3 - |-1|$

$= 1^3 - 1 = 0 \quad \therefore$ C

Q2

$= 0.0253 \quad \therefore$ D

Q3

$x(x-1) = 6$

$x^2 - x = 6$

$x^2 - x - 6 = 0.$

$(x-3)(x+2) = 0.$

$x = 3, \; x = -2 \quad \therefore$ B.

Q4

$27x^3 + 1$

$= (3x)^3 + 1^3$

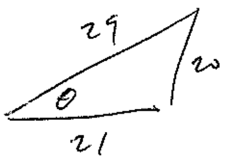
$= (3x+1)(9x^2 - 3x + 1) \quad \therefore$ D

Q5

$1 - x > 0$

$x < 1 \quad \therefore$ D

Q6



$\therefore \cos\theta = -\frac{21}{29}$

$\therefore$ B

Q7

$\sqrt{3}y = -x + 2\sqrt{3}$

$y = -\frac{1}{\sqrt{3}}x + 2.$

$\therefore \tan\theta = \frac{-1}{\sqrt{3}} \quad \therefore \theta = 150^\circ.$

$\therefore$ D.

Q8

$\log_a 18 = \log_a (3^2 \times 2)$

$= \log_a 2 + 2\log_a 3$

$= 0.4 + 2 \times 0.6$

$= 1.6 \quad \therefore$ B

Q9

$\lim_{t \to \infty} \frac{t}{2t+3}$

$= \lim_{t \to \infty} \frac{1}{2 + \frac{3}{t}} = \frac{1}{2}. \quad \therefore$ C.

Q10

$= 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

$\therefore$ C.

Q11

(a) 2.67

(b) 480°

(c) $2\sin\theta - 1 = 0$.

$\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$.

(d) $\frac{3+\sqrt{2}}{6+\sqrt{2}} \times \frac{6-\sqrt{2}}{6-\sqrt{2}}$

$= \frac{18 - 3\sqrt{2} + 6\sqrt{2} - 2}{36 - 2}$

$= \frac{16 + 3\sqrt{2}}{34}$.

(e) $|2x - 1| < 3$

$2x - 1 < 3$ & $1 - 2x < 3$

$2x < 4$ $\quad$ $2x > -2$

$x < 2$ $\quad$ $x > -1$.

$\therefore -1 < x < 2$.

(f)

(i) $y = x^2 e^x$

$u = x^2 \quad v = e^x$

$u' = 2x \quad v' = e^x$

$y' = 2xe^x + x^2 e^x$

$y' = xe^x(2 + x)$.

(f)

(ii) $y = \frac{\ln x}{x}$.

$u = \ln x \quad v = x$

$u' = \frac{1}{x} \quad v' = 1$.

$y' = \frac{1 - \ln x}{x^2}$

(g) $\int \frac{x}{x^2-1}\,dx$

$= \frac{1}{2}\int \frac{2x}{x^2-1}\,dx$

$= \frac{1}{2}\ln(x^2-1) + C$.

(h) AP series with $a = 1$

$d = 3$

$n = 11$

$S_{11} = \frac{11}{2}(1 + 31)$

$= 176$.

Q12

(a)

(i) $m_{PQ} = \frac{4-1}{-2-4} = \frac{3}{-6} = -\frac{1}{2}$.

$x+2y+2=0$.

$2y = -x-2$

$y = -\frac{1}{2}x - 1$

$\therefore m_{RS} = -\frac{1}{2}$.

(ii) $PQ^2 = (-2-4)^2 + (4-1)^2$

$PQ^2 = 36 + 9 = 45$

$\therefore PQ = 3\sqrt{5}$.

(iii) $x+2y+2=0$.

when $x=4$ $\quad 4+2y+2=0$.

$2y = -6 \quad \therefore y = -3$

$\therefore R = (4,-3)$

(iv) $h = \frac{|-2+2(4)+2|}{\sqrt{1^2+2^2}}$

$h = \frac{8}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$.

(v)

$A = \frac{1}{2}\left(\frac{8\sqrt{5}}{5}\right) \times \left(3\sqrt{5} + \sqrt{85}\right)$.

$= \frac{4\sqrt{5}}{5}\left(3\sqrt{5} + \sqrt{85}\right)$.

$= \frac{60}{5} + \frac{20\sqrt{17}}{5}$.

$= 12 + \frac{20\sqrt{17}}{5}$.

(b)

(i) $y = 8x^2 + 32x + 36$

$y - 36 = 8x^2 + 32x$.

$y - 36 = 8(x^2 + 4x)$

$y - 36 + 32 = 8(x^2 + 4x + 4)$.

$y - 4 = 8(x+2)^2$

(ii) $V = (-2, 4)$.

(iii) $\frac{1}{8}(y-4) = (x+2)^2$.

$\therefore 4a = \frac{1}{8} \quad \therefore a = \frac{1}{32}$.

$\therefore F = \left(-2, 4\frac{1}{32}\right)$

(iv) $D: y = 3\frac{31}{32}$

Q13

(a) $y = -x^3 + 12x + 1$

(i)

$y' = -3x^2 + 12$

$y'' = -6x$

$y' = 0$ when $-3x^2 + 12 = 0$.

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$.

$y(2) = 17 \quad y(-2) = -15$

$\therefore$ stat pts are $(2, 17)$ & $(-2, -15)$

$y''(2) = -12 \quad \therefore$ max at $(2, 17)$

$y''(-2) = 12 \quad \therefore$ min at $(-2, -15)$

(ii) $y'' = 0$ when $x = 0$.

$y(0) = 1$

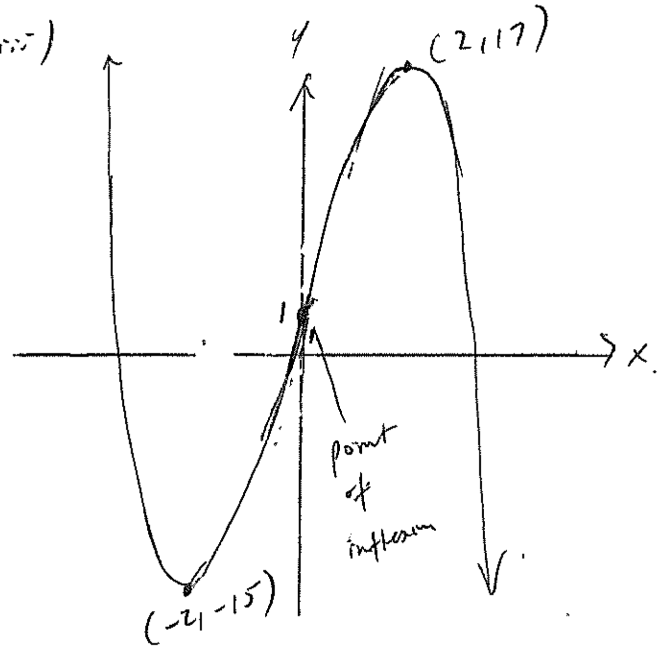
$\therefore$ possible point of inflexion at $(0, 1)$

x	-1	0	1
y''	$+$	0	$-$

$y''(-1) = 6 \quad y''(1) = -6$

$\therefore (0, 1)$ is a point of inflexion.

(iii)



(iv) $0 < x < 2$.

(b) $h = \frac{8-0}{4} = 2$.

$\therefore \int_0^8 xe^x\,dx$

$\approx \frac{2}{3}\left[f(0) + 4f(2) + 2f(4) + 4f(6) + f(8)\right]$.

$\approx \frac{2}{3}\left[0 + 4 \times 2e^2 + 2 \times 4e^4 + 4 \times 6e^6 + 8e^8\right]$

≈ 22683.9 (1dp)

Q13

(c)

(i) $a=5 \quad d=2$

$T_n = 5+(n-1)\times 2$

$T_n = 5+2n-2$

$T_n = 3+2n$

(ii) $S_n = \frac{n}{2}(a+l)$.

$140 = \frac{n}{2}(5+3+2n)$.

$140 = n(4+n)$.

$140 = 4n+n^2$

$n^2+4n-140=0$.

$n \quad +14$

$n \quad -10$

$(n+14)(n-10)=0$

$\therefore n=10$

Q14

(a)

(i) $r = \frac{10x}{5} = 2x$

$-1 < 2x < 1$

$-\frac{1}{2} < x < \frac{1}{2}$

ii $S_\infty = \frac{a}{1-r}$

$100 = \frac{5}{1-2x}$.

$20 = \frac{1}{1-2x}$.

$20-40x=1$.

$40x=19$

$x = \frac{19}{40}$.

(b)

(i) $\angle DAB = \pi - 2\times\frac{\pi}{6}$

$= \frac{2\pi}{3}$.

(ii) $BD^2 = 3^2+3^2-2\times 3^2\times \cos\frac{2\pi}{3}$.

$BD^2 = 27$

$\therefore BD = 3\sqrt{3}$.

(iii) $A = \frac{1}{2}\times 3^2\times \sin\frac{2\pi}{3} + \frac{1}{2}\times\frac{5\pi}{6}\times 27$

$A = 39.2\ m^2$ (1dp).

Q14

(c)

(i) $\angle MYX$ is common

$\angle MXY = \angle NBX$ (corresponding $\angle$'s, $BN \parallel MX$).

$\therefore \triangle XMY \parallel\!\mid \triangle BNY$ (Equiangular).

(ii) $\dfrac{MX}{NB} = \dfrac{MY}{NY}$ (ratio of matching sides of $\parallel\!\mid$ $\triangle$'s).

$\dfrac{MX}{NB} = \dfrac{1}{2}$.

$\therefore 2MX = NB$.

(iii) $\dfrac{MX}{NB} = \dfrac{1}{2}$

but $MX = MC$ (Equal sides opposite equal sides)

$\therefore \dfrac{MC}{NB} = \dfrac{1}{2}$.

Q15

(a)

(i) Let A_n be the amount owing by end of n^{th} month.

$A_1 = 2500(1.01) - 50$

$A_2 = 2500(1.01)^2 - 50(1.01) - 50$.

(ii) $A_3 = 2500(1.01)^3 - 50(1.01)^2 - 50(1.01) - 50$.

$A_3 = 2500(1.01)^3 - 50(1 + 1.01 + 1.01^2)$.

$\therefore$

$A_n = 2500(1.01)^n - 50(1 + 1.01 + 1.01^2 + \dots + 1.01^{n-1})$

$A_n = 2500(1.01)^n - 50\left(\dfrac{1.01^n - 1}{0.01}\right)$.

$A_n = 2500(1.01)^n - 5000(1.01)^n + 5000$

$A_n = 5000 - 2500(1.01)^n$

$A_n = 0$ when

$0 = 5000 - 2500(1.01)^n$

$2500(1.01)^n = 5000$

$(1.01)^n = 2$

$n \ln(1.01) = \ln 2$

$n = \dfrac{\ln 2}{\ln 1.01} = 70$ months.

(ii). Interest $= 70 \times 50 - 2500$

$= \$1000$

(iii)

$A_n = 2500(1.0075)^n - 50\left(\dfrac{1.0075^n - 1}{0.0075}\right)$.

$A_n = 0$ when

$0 = 2500(1.0075)^n - 50\left(\dfrac{1.0075^n - 1}{0.0075}\right)$

$0 = 18.75(1.0075)^n - 50(1.0075^n - 1)$

$0 = 50 - 31.25(1.0075)^n$

$(1.0075)^n = \dfrac{50}{31.25}$

$n \ln 1.0075 = \ln \dfrac{50}{31.25}$.

$n = \dfrac{\ln \frac{50}{31.25}}{\ln 1.0075} = 63$ months.

4/5

(b)

(i) when $t=4$ $P=100$

$$100 = 50e^{4.k}$$

$$e^{4k} = 2.$$

$$4k = \ln 2$$

$$k = \frac{\ln 2}{4}.$$

(ii) $t=10$ $P = 50e^{10k} = 282$

(iii) $\frac{dP}{dt} = 50ke^{kt}.$

when $t=10$ $\frac{dP}{dt} = 50ke^{10k}.$

$= 49$ insects per day.

(iv) $1000 = 50e^{kt}.$

$$e^{kt} = 20$$

$$kt = \ln 20.$$

$$t = \frac{\ln 20}{k} = 17.28\cdots \text{ days}$$

or 18 days (whole day).

(c)

(i) $\sqrt{x-5} = \frac{1}{5}(x-5).$

$$\frac{1}{5}(x-5) - \sqrt{x-5} = 0.$$

$$\frac{1}{5}\sqrt{x-5}\left[\sqrt{x-5} - 5\right] = 0.$$

$\therefore x = 5$ or $\sqrt{x-5} = 5$

$$x-5 = 25$$

$$x = 30.$$

(ii)

$$\int_5^{30} \sqrt{x-5} - \frac{1}{5}(x-5)\,dx.$$

$$= \int_5^{30} (x-5)^{1/2} - \frac{1}{5}x + 1\,dx.$$

$$= \left[\frac{2}{3}(x-5)^{3/2} - \frac{1}{10}x^2 + x\right]_5^{30}$$

$$= \frac{2}{3}(25)^{3/2} - \frac{30^2}{10} + 30 - \left(-\frac{25}{10} + 5\right)$$

$$= \frac{125}{6}.$$

a16

(a)

(i) $\theta R = 2\pi r$

$\therefore r = \frac{R\theta}{2\pi}$

(ii) $H^2 = R^2 - r^2$

$H^2 = R^2 - \frac{R^2\theta^2}{4\pi^2}$

$H^2 = \frac{R^3}{4\pi^2}\left(4\pi^2 - \theta^2\right)$.

(iii) $V = \frac{1}{3}\pi r^2 H$.

$V = \frac{1}{3}\pi \times \frac{R^2\theta^2}{4\pi^2} \times \frac{R}{2\pi}\sqrt{4\pi^2 - \theta^2}$

$V = \frac{R^3\theta^2}{24\pi^2}\sqrt{4\pi^2 - \theta^2}$

(iv) $u = \frac{R^3\theta^2}{24\pi^2}$ $\quad v = \sqrt{4\pi^2 - \theta^2}$

$v = (4\pi^2 - \theta^2)^{1/2}$

$u' = \frac{R^3\theta}{12\pi^2}$ $\quad v' = \frac{1}{2}(4\pi^2 - \theta^2)^{-1/2} \times -2\theta$.

$v' = -\frac{\theta}{\sqrt{4\pi^2 - \theta^2}}$

$\frac{dV}{d\theta} = \frac{R^3\theta}{12\pi^2}\sqrt{4\pi^2 - \theta^2} - \frac{R^3\theta^3}{24\pi^2\sqrt{4\pi^2 - \theta^2}}$

$\frac{dV}{d\theta} = \frac{2R^3\theta(4\pi^2 - \theta^2) - R^3\theta^3}{24\pi^2\sqrt{4\pi^2 - \theta^2}}$

$\frac{dV}{d\theta} = \frac{8\pi^2R^3\theta - 2R^3\theta^3 - R^3\theta^3}{24\pi^2\sqrt{4\pi^2 - \theta^2}}$.

$\frac{dV}{d\theta} = \frac{8\pi^2R^3\theta - 3R^3\theta^3}{24\pi^2\sqrt{4\pi^2 - \theta^2}}$

$\frac{dV}{d\theta} = 0$ when $8\pi^2R^3\theta - 3R^3\theta^3 = 0$

$\therefore R^3\theta(8\pi^2 - 3\theta^2) = 0$.

$\therefore 8\pi^2 - 3\theta^2 = 0$

$\therefore 3\theta^2 = 8\pi^2$

$\theta^2 = \frac{8\pi^2}{3}$

$\therefore \theta = \frac{2\sqrt{2}\pi}{\sqrt{3}}$ as $\theta > 0$.

(b)

(i) $e^{-t} > 0$

So $v < 0$ when $t - 3 < 0$

$\therefore t < 3$.

(ii) $u = 2 - t$. $\quad v = e^{-t}$.

$u' = -1 \quad v' = -e^{-t}$.

$\frac{d}{dt}(2-t)e^{-t} = -e^{-t} - (2-t)e^{-t}$.

$= -e^{-t} - 2e^{-t} + te^{-t}$.

$= -3e^{-t} + te^{-t}$.

$= (t-3)e^{-t}$.

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(b)

(iii)

$$D(T) = -\int_0^3 v\,dt + \int_3^T v\,dt.$$

$$= \Big[(t-2)e^{-t}\Big]_0^3 + \Big[(2-t)e^{-t}\Big]_3^T$$

$$= e^{-3} + 2 + (2-T)e^{-T} + e^{-3}$$

$$= 2 + \frac{2}{e^3} + \frac{2-T}{e^T}$$

(iv) $T \to \infty$ $D(T) \to 2 + \frac{2}{e^3}$.

(c)

(i) 1000

(ii) $\alpha + \beta = \alpha\beta$

$$-\frac{b}{a} = \frac{c}{a}$$

$\therefore -b = c$.

$\therefore$ 10 options for a

10 options for b

Once b is choosen, c has to $-b$

$$\therefore p = \frac{10 \times 10}{1000} = \frac{1}{10}.$$

(iii).

Equal roots implies that

$\Delta = b^2 - 4ac = 0$.

i.e $b^2 = 4ac$

So the square of b has to be a multiple of 4.

So $b = \pm 2$ or $b = \pm 4$.

Furthermore $4ac = b^2$

So $ac > 0$ $\therefore$ a & c must have the same sign.

So if $b = \pm 2$ then $ac = 1$ so

$a = 1$ $c = 1$

or

$a = -1$ $c = -1$.

if $b = \pm 4$ then $ac = 4$ so

$a = 1$ $c = 4$ $a = -1$ $c = -4$

$a = 4$ $c = 1$ $a = -4$ $c = -1$.

$a = 2$ $c = 2$ $a = -2$ $b = -2$.

$$\text{So } p = \frac{4 + 12}{1000} = \frac{2}{125}.$$